1st Sem Digital Electronics

USN | | | | | | | | | | |

12EC046

M.Tech. Degree Examination, February 2013
Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Obtain the row reduced echelon form of the matrix

$$A = \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix}$$

Locate the Pivot column of A and Pivotal position of A. **(06 Marks)**

b. Find the general solution of the linear system of equation whose augmented matrix has been reduced to

$$\begin{bmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

(07 Marks)

c. Find the inverse of the matrix.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$$

(07 Marks)

2 a. Define vector space and sub space and also prove that a non empty sub space W of a vector space V over a field F is a subspace of V if and only if, W is closed under vector addition and scalar multiplication. **(10 Marks)**

b. Find the dimension and basis of the sub space spanned by the vectors (2, 4, 2) , (1, -1, 0) , (1, 2, 1) and (0, 2, 1) in $V_3(R)$. **(10 Marks)**

3 a. Show that the vectors (1, 1, 2, 4) , (2, -1, -5, 2) , (1. -1, -4, 0) and (2, 1, 1, 6) are linearly independent R^4 and extract a linearly independent subset. **(10 Marks)**

b. Given the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$, determine the linear transformation $T : V_3(R) \rightarrow V_2(R)$ relative to bases. $B_1 = \{(1, 1, 1) , (1, 2, 3), (1, 0, 0)\}$ and $B_2 = \{(1, 1) , (1, -1)\}$. **(10 Marks)**

4 a. If V is an n – dimensional vector space over a field F and W is an m – dimensional vector space over F, then show that the space L(V, W) is finite dimensional and has dimension mn. **(10 Marks)**

b. Let V and W be finite dimensional vector spaces over the field F such that dim V = dim W. If T is a linear transformation from V into W, then prove that the following are equivalent
i) T is invertible ii) T is non – singular iii) T is onto, that is, the range of T is W. **(10 Marks)**

5 a. State and prove primary decomposition theorem. **(10 Marks)**

b. Explain the following and illustrate with an example : i) Annihilating polynomial ii) Minimal polynomial iii) Invariant subspace iv) Direct sum decomposition. **(10 Marks)**

6 a. Use Gram – Schmidt process on the basis {(1, 0, 0), (1, 1, 0), (1, 1, 1)} to obtain orthonormal basis of R^3. **(10 Marks)**

b. Find a least square solution of system represented by Ax = b, where

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, \; b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$ **(10 Marks)**

7 a. Diagonalise the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$. **(10 Marks)**

b. Find a QR factorization of

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$ **(10 Marks)**

8 a. Find the maximum and minimum values of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$ Subject to the constraints $X^T X = 1$. **(10 Marks)**

b. Find the singular value decomposition of the matrix.

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}.$$ **(10 Marks)**

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